

# Automata-Based Temporal Reasoning in Answer Set Programming with Application to Process Mining

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- Show how to perform temporal reasoning in ASP using automata;
- Apply the method to Declarative Process Mining;
- Problems considered: Log Generation, Conformance Checking, and Query Checking.

- Process Mining (PM) is at the intersection of Business Process Management and Data Mining;
- PM analyzes event logs to extract information about the underneath process.
- Process models are typically Petri nets<sup>1</sup> or Business Process Modeling Notation<sup>2</sup>.

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<sup>1</sup>Wil M. P. van der Aalst. "The Application of Petri Nets to Workflow Management". In: *J. Circuits Syst. Comput.* 8.1 (1998), pp. 21–66

<sup>2</sup>Stephen A. White and Conrad Bock. *BPMN 2.0 Handbook Second Edition*. Future Strategies Inc., 2011

# Declarative Process Mining

- Declarative PM specifies processes in a constraint-based fashion
- Formalisms used are DECLARE<sup>3</sup>, LTL<sub>f</sub><sup>4</sup>, and LTL<sub>p</sub><sup>5</sup>;
- In DPM models specify the properties of the (traces of the) process
  - it specify *what* property a trace should have, rather than *how* to construct them
  - reduces false negative (i.e., traces erroneously excluded by the model)

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<sup>3</sup>Wil M. P. van der Aalst, Maja Pesic, and Helen Schonenberg. “Declarative workflows: Balancing between flexibility and support”. In: *Comput. Sci. Res. Dev.* 23.2 (2009), pp. 99–113

<sup>4</sup>Giuseppe De Giacomo and Moshe Y. Vardi. “Linear Temporal Logic and Linear Dynamic Logic on Finite Traces”. In: *Proc. of the 23rd Int. Joint Conf. on Artificial Intelligence. IJCAI/AAAI, 2013*

<sup>5</sup>Valeria Fionda and Gianluigi Greco. “LTL on Finite and Process Traces: Complexity Results and a Practical Reasoner”. In: *J. Artif. Intell. Res.* 63 (2018), pp. 557–623

- **Log generation**: generate a log compliant with a process model.
- **Conformance checking**: check whether the traces are compliant with a process model.
- **Query checking**: finding properties of a process by checking possible templates against the event log of the process.

- Given a set  $\mathcal{P}$  of propositional symbols, the syntax is defined by the following grammar:

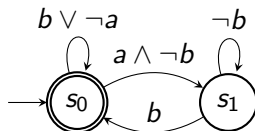
$$\varphi ::= A \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathbf{X}\varphi \mid \varphi_1 \mathbf{U}\varphi_2$$

with  $A \in \mathcal{P}$ .

- Common abbreviations used are:
  - $\text{true}$ ,  $\rightarrow$ ,  $\vee$
  - $\mathbf{F}\varphi \equiv \text{true}\mathbf{U}\varphi$
  - $\mathbf{G}\varphi \equiv \neg\mathbf{F}\neg\varphi$
  - $\varphi_1\mathbf{W}\varphi_2 \equiv \varphi_1\mathbf{U}\varphi_2 \vee \mathbf{G}\varphi_1$

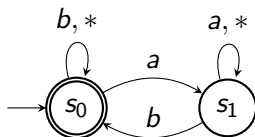
- Given a formula  $\varphi$ , a trace  $\pi = \pi_1, \pi_2, \dots, \pi_{len(\pi)} \in (2^{\mathcal{P}})^+$ , and a time instant  $i$ , with  $1 \leq i \leq len(\pi)$ , the semantics is defined as follows:
  - $\pi, i \models A$  iff  $A \in \pi_i$ ,
  - $\pi, i \models \neg\varphi$  iff  $\pi, i \not\models \varphi$ ,
  - $\pi, i \models \varphi_1 \wedge \varphi_2$  iff  $\pi, i \models \varphi_1$  and  $\pi, i \models \varphi_2$ ,
  - $\pi, i \models \mathbf{X}\varphi$  if  $i < len(\pi)$  and  $\pi, i + 1 \models \varphi$ ,
  - $\pi, i \models \varphi_1 \mathbf{U}\varphi_2$  iff  $\pi, j \models \varphi_2$  for some  $j$ , with  $i \leq j \leq len(\pi)$ , and  $\pi, k \models \varphi_1$  for all  $k = i, \dots, j - 1$ .
- A formula  $\varphi$  is true in  $\pi$ , and we write  $\pi \models \varphi$ , if  $\pi, 1 \models \varphi$ .

- For each LTL<sub>f</sub> formula  $\varphi$  there exists a NFA  $A_\varphi$  that accepts exactly the traces that satisfy  $\varphi$ .
- For example to  $\varphi = \mathbf{G}(a \rightarrow \mathbf{F}b)$  is associated





- $LTL_p$  restrict the semantics to consider only process traces (or simple finite traces)
- This, in turn, result in simpler automata where arcs are labeled directly by activities<sup>6</sup>



<sup>6</sup>Francesco Chiariello, Fabrizio Maria Maggi, and Fabio Patrizi. “From LTL on Process Traces to Finite-state Automata”. In: *BPM (Demos / Resources Forum)*. Vol. 3469. CEUR Workshop Proceedings. CEUR-WS.org, 2023, pp. 127–131

# Answer Set Programming

- Answer Set Programming (ASP): declarative approach for search and optimization problems <sup>78</sup>.
- Provide a modeling language for writing logic programs.
- Programs' models are computed with ASP systems such as
  - *clingo*<sup>9</sup>
  - DLV <sup>10</sup>

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<sup>7</sup>Ilkka Niemelä. “Logic Programs with Stable Model Semantics as a Constraint Programming Paradigm”. In: *Ann. Math. Artif. Intell.* 25.3-4 (1999), pp. 241–273

<sup>8</sup>Victor W. Marek and Miroslaw Truszczyński. “Stable Models and an Alternative Logic Programming Paradigm”. In: *The Logic Programming Paradigm. Artificial Intelligence*. Springer, 1999, pp. 375–398

<sup>9</sup>Martin Gebser et al. “Multi-shot ASP solving with clingo”. In: *Theory Pract. Log. Program.* 19.1 (2019), pp. 27–82

<sup>10</sup>Mario Alviano et al. “The ASP System DLV2”. In: *LPNMR*. vol. 10377. Lecture Notes in Computer Science. Springer, 2017, pp. 215–221

- A **normal rule** is of the form

$$h \leftarrow b_1, \dots, b_m, \text{not } b_{m+1}, \dots, \text{not } b_n$$

where  $h, b_1, \dots, b_n$  are atoms.

- An **integrity constraint** is of the form

$$\leftarrow b_1, \dots, b_m, \text{not } b_{m+1}, \dots, \text{not } b_n$$

- A **choice rule** with cardinality constraints is of the form

$$l\{h_1, \dots, h_n\}u$$

with  $l, u \in \mathbb{N}, l \leq u \leq n$ .

Given a logic program  $\Pi$  and a set  $X$  of atoms we define the reduct  $\Pi^X$  of  $\Pi$  w.r.t.  $X$  as the program obtained from  $\Pi$  as follows:

- if a rule contains in the negative body an atom that is in  $X$  we remove the rule,
- of the remaining rules, we remove the negative body.

In this way the resulting program  $\Pi^X$  doesn't contain default negation.  $X$  is then an *answer set*, or *stable model*, of  $\Pi$  if it coincides with the (unique) minimal model of  $\Pi^X$ .

- Generate and test (also called Guess and Check) methodology:
  - 1 Generate: guess a candidate solution
  - 2 Test: check if the candidate is a proper solution
- Differences from brute force:
  - candidate's selection
  - evaluation of partial candidates

The proposed approach<sup>11</sup> consists of the following steps:

- Convert temporal specifications to automata.
- Represent automata in ASP.
- Represent traces in ASP.
- Modeling how automata read trace.
- Add generation and test rules.

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<sup>11</sup>Francesco Chiariello, Fabrizio Maria Maggi, and Fabio Patrizi. “ASP-Based Declarative Process Mining”. In: *AAAI*. AAAI Press, 2022, pp. 5539–5547

Predicates:

- $trace(A, T)$ : activity  $A$  happens at time  $T$ .

## Example

Trace  $\pi = a_2, a_1, a_2$  becomes:

- $trace(a_2, 1)$ .
- $trace(a_1, 2)$ .
- $trace(a_2, 3)$ .

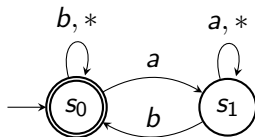
- $init(S)$ :  $S$  is the initial state.
- $acc(S)$ :  $S$  is an accepting state.
- $trans(S, F, S')$ : there exists a transition from state  $S$  to state  $S'$  labeled with event formula  $F$ .
- $holds(F, T)$ : event formula  $F$  holds at time  $T$ .



# Example

The ASP encoding of the  $LTL_p$  formula  $\varphi = \mathbf{G}(a \rightarrow \mathbf{F}b)$  is given by:

- $init(s_0)$ .
- $acc(s_0)$ .
- $trans(s_0, 1, s_1)$ .
- $holds(1, T) \leftarrow trace(a, T)$ .
- $trans(s_1, 2, s_0)$ .
- $holds(2, T) \leftarrow trace(b, T)$ .
- $trans(s_0, 3, s_0)$ .
- $holds(3, T) \leftarrow trace(b, T)$ .
- $holds(3, T) \leftarrow trace(C, T), C \neq a, C \neq b$ .
- $trans(s_1, 4, s_1)$ .
- $holds(4, T) \leftarrow trace(A, T), A \neq b$ .



Predicate *state* models execution of automaton on trace

- $state(S, T)$ :  $S$  is current state at time  $T$ .

and updated as

- $state(S, 0) \leftarrow init(S)$ .
- $state(S', T) \leftarrow state(S, T - 1), trans(S, F, S'), holds(F, T)$ .

Given an formula and trace length  $t$ ,

Generate traces as follows

- $\{trace(A, T) : activity(A)\} = 1 \leftarrow time(T)$ .

Test traces as follows

- $sat \leftarrow state(S, t), accepting(S)$ .
- $\leftarrow not sat$ .

- Traces are given as input
- Just check whether they are accepted

# Query Checking Example

**Query checking:** finding properties of a process by checking possible templates against its event log.

- Input
  - Log: (a,b,c,c,b); (c,b,c,c,c)
  - Formula:  $\mathbf{G}(?a \rightarrow \mathbf{F}?b)$
  - (optional) Constraints number: 1
- Output:  $\mathbf{G}(a \rightarrow \mathbf{F}b)$

The following predicates are introduced

- $var(V)$ :  $V$  is a variable.
- $assgnmt(V, A)$ : activity  $A$  is assigned to variable  $V$ .

The body of the rule for *holds* is modified by replacing  $trace(act, T)$  with  $trace(A, T)$ ,  $assgnmt(v, A)$ , with  $v$  being the variable in place of activity *act*.

Then for generating

- $\{assgnmt(V, A) : activity(A)\} = 1 \leftarrow var(V)$ .

and for testing we check that the formula is satisfied by the trace.

- We have seen how the automata representation of temporal specifications can be used in ASP to perform temporal reasoning.
- We have considered Declarative Process Mining as an application domain to illustrate the approach.
- The contributions are manifold and benefit different communities:
  - To the Temporal Logics community, it provides a tool to perform temporal reasoning;
  - To the ASP community, it provides a method to intuitively handle time;
  - to the Process Mining community, it provides both tools and methods for analyzing event logs.

- Application to other DPM problems, e.g.,
  - Process Discovery,
  - Process Model Repair,
  - Trace Alignment.
- Application to other areas, e.g.
  - Discrete Event Systems,
  - Planning,
  - **Put your field here.**



Thank you!!

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